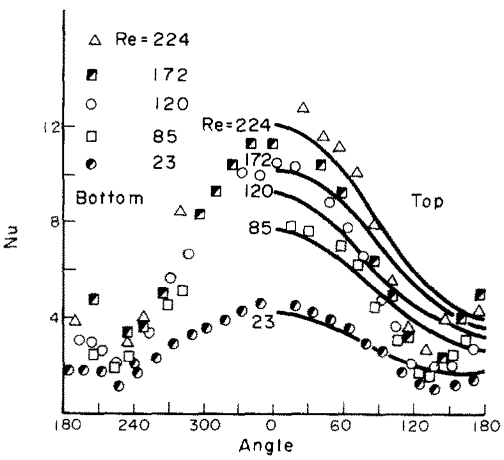

FIG. 2. Heat transfer around a circular cylinder to a cross-flow of air.

APPLICATION IN HOT FILM ANEMOMETRY

For measurements of turbulence, the local slope of the calibration curve of a hot wire anemometer is commonly used to compute such quantities as turbulence intensities, Reynolds stresses, etc. The calibration curve is fitted to the calibration data either graphically or with the help of a statistical criterion. The latter involves the finding of at least two out of the three constants, A, B, and n, of the widely used empirical formula

$$Nu = A + B \cdot Re^n. \tag{7}$$

Even so, it is doubtful that the fitted equation gives the correct first deviatives over the entire range of interest. Therefore, the graphical method is generally favored. However, the accuracy of the graphical method depends heavily on the individual's personal judgment and experience; moreover, the graphical method cannot be used to evaluate a large amount of data with the help of a computer. For computer calculations, a theoretical equation representing the essential behavior of the hot wire anemometer, with as few undetermined local constants as possible, is needed. It is believed that the present theory through equation (6) satisfies this need.

The deviation of the calibration curve of a hot wire anemometer from time to time is generally attributed to the contamination of impurities on the wire surface. It can be regarded as a coating of certain thickness. In equation (6), ε, b and A are given by equations (2), (3) and (5). The local constant C can be found by best fit of equation (6) to the calibration data.

This method has been actually used to process the data obtained in the mercury channel described in [2]. The results were comparatively better than those obtained with the statistical method.

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EFFECT OF STALL LENGTH ON HEAT TRANSFER IN REATTACHED REGION BEHIND A DOUBLE STEP AT ENTRANCE TO AN ENLARGED FLAT DUCT

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NOMENCLATURE

- h, step height;
- L, entrance height;
- $Nu_{d,\,max}$, maximum value of local Nusselt number based on hydraulic diameter;
- $Nu_{L,\,max}$, maximum value of local Nusselt number, $\alpha_{max}L/\lambda$;
- Re_d, Reynolds number based on hydraulic diameter;
- Re_L, Reynolds number, uL/ν;
- q_w, wall heat flux;
- u, flow velocity at entrance;
- x_R, overall stall length.

Greek symbols

- α, heat-transfer coefficient;
- λ, thermal conductivity;
- ν, kinematic viscosity.

INTRODUCTION

IN A FLOW region with an abruptly enlarged area change of a tube or of a duct, it is well known that the separated and the reattached regions occur. Especially, heat-transfer problems for such a flow geometry have been studied by several investigators. For a circular cross-sectional duct, Krall and Sparrow [1] and Ede *et al.* [2] have reported their results with water under the condition of constant heat flux. On the other hand, for a rectangular cross-sectional duct, Filetti and Kays [3] have performed their experiments with air under the condition of constant wall temperature for two test configurations, $h/L = 0.5625$ and $h/L = 1.05$. They reported that the heat-transfer rate at a reattachment point was dependent on step height ratio. However, the applicability of their proposed empirical relations describing $Nu_{d,\max}$ vs h/L for a wide range of step height has not yet been clarified.

The purpose of this investigation is to determine the effect of step height on heat-transfer rate at a reattachment point, for a rectangular cross-sectional duct. Especially, the maximum Nusselt number is discussed by correlating it with a stall length. The flow geometry is shown in Fig. 1, and the test fluid is air. The heat-transfer characteristics at the reattachment point are examined for seventeen kinds of step height ratio h/L, which is varied between 0.035 and 7.0, under the condition of constant heat flux. Reynolds number ranges approximately from 4×10^3 to 8×10^4.

EXPERIMENTAL APPARATUS

Essential components of the apparatus consist of a contraction, steps and heated plates, which are installed in the test section of an open circuit tunnel. The cross-sectional area and the length of the test section are 150 × 150 mm and 600 mm, respectively. Flow velocity at the entrance of the test section is measured by a Chattock manometer connected with a Pitot-static tube. Heating of the plates is accomplished by applying an electric current to thirty-six parts of nichrome wire embedded in the plates, thus producing a uniform heat flux over the plates. Wall temperatures are measured by 0.3 mm dia C–C thermocouples situated at 24 longitudinal stations. Assuming that the maximum heat transfer is to occur at the reattachment point, stall length is estimated from the temperature distribution of the wall. Two-dimensionality of flow in the reattachment region is examined with thermocouples and confirmed by a flow visualization using the oil film method.

RESULTS AND CONCLUSIONS

The heat-transfer coefficient in this note is defined as the following expression on account of the difficulty of measuring a bulk temperature in a separated region.

$$\alpha = q_w/(t_w - t_c) \tag{1}$$

where t_w and t_c are the wall and the center-line fluid temperatures, respectively.

Krall and Sparrow obtained a relation as $Nu_{d,\max} \sim Re_d^{2/3}$, where $Nu_{d,\max}$ and Re_d are based on a hydraulic diameter of d, for a tube flow. On the other hand, for a rectangular duct flow, Filetti and Kays presented two relations as $Nu_{d,\max} \sim Re_d^{0.689}$ for a short stall and as $Nu_{d,\max} \sim Re_d^{0.593}$ for a long stall. But, if the discussion is focussed only on the effect of step height on maximum Nusselt number, one might realize the exponent of Reynolds number was independent of step height. For instance, a relation between maximum Nusselt number and step height is given by Filetti and Kays for a short stall as

$$Nu_{d,\max} = [0.124 + 0.101(2h/L)]\, Re_d^{0.689}. \tag{2}$$

Dividing equation (2) by $Re_d^{2/3}$, the following expression is obtained.

$$Nu_{d,\max}/Re_d^{2/3} = [0.124 + 0.101(2h/L)]\, Re_d^{0.022}. \tag{3}$$

From this expression, it could be easily understood that the slope of $Nu_{d,\max}/Re_d^{2/3}$ against $(2h/L)$ is not affected by the exponent of Reynolds number in case of logarithmic plotting. In this note, the exponent of Reynolds number is adopted as 2/3.

Figure 1 shows $Nu_{L,\max}$ obtained for $h/L = 0.035 \sim 7.0$. However, for the long stall, data are confined in a relatively small range due to the length restriction of the experimental apparatus. From these results, it can be seen that the maximum heat-transfer rate for a short stall is greater than that for a long stall. Further, the deviation between both the maximum heat-transfer rates becomes larger with increasing step height. As the experimental conditions performed by Filetti and Kays are different from authors', the comparison between both results may be impossible in the strict sense. As a reference, only the slopes of their data are shown in Fig. 1. As will be seen in this figure, the slopes of their data agree well with the present results for $0.5625 < h/L < 1.05$. However, for a wider range of h/L, it is clear that their extrapolated predictions shown in Fig. 1 as dotted lines are not enough. These discrepancies seem to be resulted from an assumption in their work that maximum Nusselt number is to vary linearly with step height. Consequently, it may be natural that a linear relationship between maximum Nusselt number and step height does not hold good in a wider range of h/L.

It is ascertained that the stall lengths obtained in the present investigation agree well with the results for double step configurations by Abbott and Kline [4]. The comparison of these results with those in Fig. 1 clarifies the fact that there is a similar tendency between the maximum heat-transfer rates against step height and the stall lengths against it.

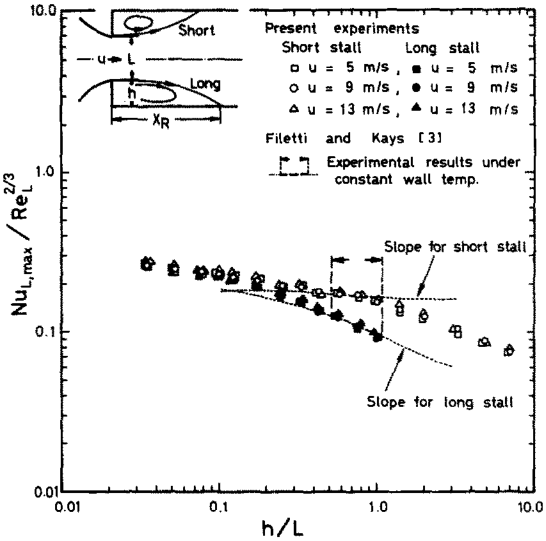

FIG. 1. Correlation between step height and maximum Nusselt number.

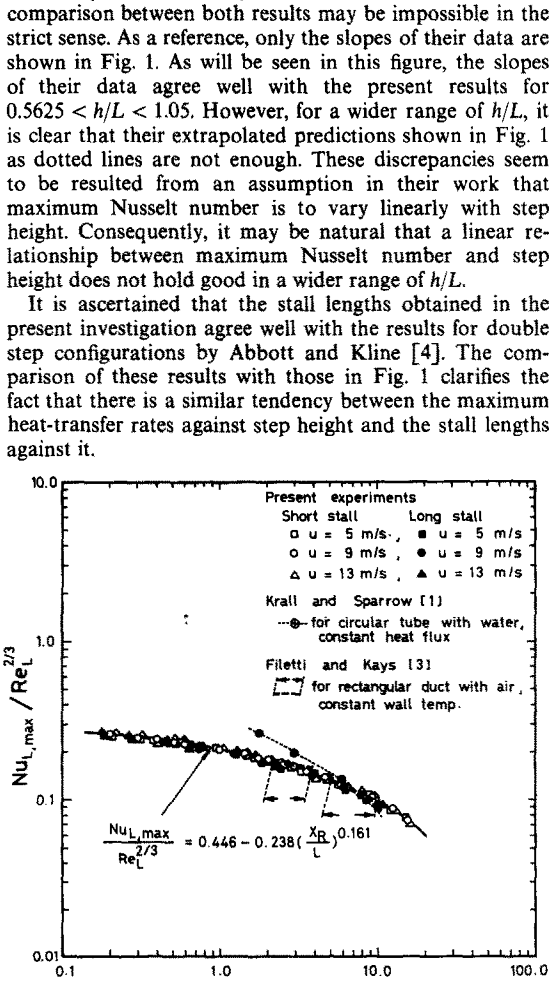

FIG. 2. Correlation between stall length and maximum Nusselt number.

The aforementioned experimental results suggest that $Nu_{L,\max}$ might be related with stall length. Figure 2 shows the plotted result of $Nu_{L,\max}/Re_L^{2/3}$ vs x_R/L. It may be concluded that the data are well correlated by the following expression.

$$Nu_{L,\max} = [0.446 - 0.238(x_R/L)^{0.161}]\, Re_L^{2/3}. \quad (4)$$

In Fig. 2, the experimental data by Krall *et al.* and Filetti *et al.* are compared with authors' results. The slopes of Filetti's results show a good agreement with authors' in the limited range of x_R/L. It is interesting to note that $Nu_{L,\max}$ is approximated by (4) without any distinction of short or long stall. In conclusion, $Nu_{L,\max}$ is solely dependent on a stall length, x_R/L, and decreases with increasing stall length. The accuracy of $Nu_{L,\max}$ estimated by the empirical relation (4) is within $\pm 10\%$ for $4\times10^3 < Re_L < 8\times10^4$ and $0.2 < x_R/L < 16.0$.

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AN INTEGRAL EQUATION APPROACH TO AC DIFFUSION

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NOMENCLATURE

T,	unknown function;
D,	diffusion constant;
i,	imaginary unit;
ω,	angular frequency;
G,	Green's function;
r,	distance in the two dimensional plane;
ker_0, kei_0	Kelvin functions of order zero;
$\bar{u}_n$,	unity vector normal to the boundary;
ρ,	unknown boundary function.

1. INTRODUCTION

Whereas the integral equation technique has been widely used for potential and electromagnetic scattering problems [1–8], this technique is not commonly known for other applications. The basic idea for using an integral equation consists in the numerical solution of the problem. A two dimensional partial differential equation will be replaced by a one dimensional integral equation. This fact saves memory storage and computation time. The programming of the problem is then also simplified.

The transient analysis of a thermal diffusion problem by an integral equation has been performed by Shaw [9]. Similar methods have been applied for a drift-diffusion problem [10, 11]. In this paper, an integral equation will be derived for the equation:

$$D\nabla^2 T = i\omega T \quad (1)$$

which describes the diffusion phenomenon in a two dimensional area S under AC conditions ($\partial/\partial t \to i\omega$).

2. INTEGRAL EQUATION

In order to establish an integral equation for the equation (1), one has to know the Green's function G of the problem. This function is a solution of:

$$\nabla^2 G - \frac{i\omega}{D} G = \delta(\bar{r}) \quad (2)$$

in the infinite two dimensional plane. One can then use polar coordinates (r, θ) and by taking the circular symmetry into account, the θ-dependence may be dropped. The Green's function G depends only upon the distance r and is found to be:

$$G(r) = \frac{1}{2\pi}\{ker_0[r\sqrt{(\omega/D)}] + i\,kei_0[r\sqrt{(\omega/D)}]\} \quad (3)$$

where ker_0 and kei_0 are the Kelvin functions of zeroth order [12].

The integral equation technique will now be outlined for the particular geometry presented on Fig. 1. The same method can be applied for arbitrary geometries. The boundary conditions are (Fig. 1):

$$\begin{aligned} T &= T_0 && \text{on } AA' \\ T &= 0 && \text{on } BB' \\ \nabla T\cdot \bar{u}_n &= 0 && \text{on } AB \text{ and } A'B'. \end{aligned} \quad (4)$$

By using the y-independence of this problem the equation (1) can also be solved analytically, so that the numerical results can be compared with the exact analytical solution. In order to construct the integral equation, the solution T is written as:

$$T(\bar{r}) = \oint_C \rho(\bar{r}')G(|\bar{r}-\bar{r}'|)\,dC' \quad (5)$$

where $\rho(\bar{r})$ is an unknown complex source function defined along the boundary C. Imposing the boundary conditions (4) on the proposed solution (5) yields:

$$\oint_C \rho(\bar{r}')G(|\bar{r}-\bar{r}'|)\,dC' = T_0 \quad \bar{r}\in AA' \quad (6)$$

$$\oint_C \rho(\bar{r}')G(|\bar{r}-\bar{r}'|)\,dC' = 0 \quad \bar{r}\in BB' \quad (7)$$

$$\frac{\rho(\bar{r})}{2} + \oint \rho(\bar{r}')\nabla_{\bar{r}}\, G(|\bar{r}-\bar{r}'|)\cdot \bar{u}_n\, dC' = 0 \quad r\in AB \text{ and } A'B' \quad (8)$$